

**NARROW WIDTH PENTAQUARKS****F. Buccella** <sup>1</sup>

and

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CH - 1211 Geneva 23, Switzerland**ABSTRACT**

A general study of pentaquarks built with four quarks in a  $L = 1$  state and an antiquark in  $S$ -wave shows that several of such states are forbidden by a selection rule, which holds in the limit of flavour symmetry, to decay into a baryon and a meson final state. We identify the most promising  $\overline{10}$  multiplet for the classification of the  $\Theta^+$  and  $\Xi^{--}$  particles recently discovered with the prediction of a narrow width for both of them.

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A narrow KN resonance, called  $\Theta^+$ , has been found at 1540 MeV in different experiments [1]: it seems to have  $I = 0$ , since no similar state has been found in  $K^+p$  scattering. More recently, at CERN, a  $\Xi^{--}$  state and a  $\Xi^0$  state have been found at a mass of 1862 MeV with a width below the detector resolution of 18 MeV [2]. Since a  $\Xi^+$  is expected to exist on the basis of isospin invariance, one may conclude that all the exotic states (impossible to build with three quarks) of a flavour  $SU(3)_F$   $\overline{10}$  representation have been found. Interestingly enough, a  $\overline{10}$  of  $SU(3)_F$  with  $J^P = 1/2^+$  is predicted in the framework of the Skyrme model [3], in the same group of states of the better-established  $(8, 1/2)^+$  and  $(10, 3/2)^+$  traditionally classified in the 56-dimensional representation of flavour-spin  $SU(6)_{FS}$  [4], and the value of the mass of the  $Y = 2, I = 0$  state happened to be predicted at the right value [5]. In fact, one of the authors (D.D.) has been very active in promoting the experimental search for that state. These states can be thought to be pentaquarks, consisting of four quarks and one antiquark, with  $\Theta^+$  being a  $uudd\bar{s}$  state and the  $\Xi^{--}$  resonance a  $ddss\bar{u}$  state. Hereafter, we propose to construct the wave functions of  $4q$  states, consistent with the total antisymmetry dictated by Fermi-Dirac statistics, in colour and other degrees of freedom. In order to constitute a  $SU(3)$  colour singlet together with the antiquark, the four-quark state should transform as a 3 of  $SU(3)_c$ , corresponding to the Young tableau [2,1<sup>2</sup>]. Therefore, to build a complete antisymmetric wave function, the symmetry prescription with respect to the other variables of the four quarks should correspond to the Young tableau [3,1]. Should their spatial wave function be totally symmetric in the absence of an orbital angular momentum ( $\vec{L} = 0$ ), they have to transform as the 210 representation of flavour spin  $SU(6)_{FS}$ , which has the symmetry properties of the just-mentioned Young tableau. The corresponding pentaquark would then be constructed by composing the  $SU(3)_F \times SU(2)_S$  states of the 210 with the  $\bar{6} = (\bar{3}, 2)$  antiquark and with the orbital momentum of the  $\bar{q}$  with respect to the four-quark system: if this last one is zero –  $\bar{q}$  in a  $S$ -wave – one should get negative parity states. To get positive parity states with an  $S$ -wave  $\bar{q}$ , one could consider, as in [6] [7],  $L = 1$  four-quark states. To this extent we write the identity:

$$\begin{aligned} \sum_{i=1}^4 \vec{r}_i \wedge \vec{p}_i &= \frac{1}{4} \left\{ \left( \sum_{i=1}^4 \vec{r}_i \right) \wedge \left( \sum_{i=1}^4 \vec{p}_i \right) + (\vec{r}_1 + \vec{r}_2 - \vec{r}_3 - \vec{r}_4) \wedge (\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \right. \\ &\quad + (\vec{r}_1 + \vec{r}_3 - \vec{r}_2 - \vec{r}_4) \wedge (\vec{p}_1 + \vec{p}_3 - \vec{p}_2 - \vec{p}_4) \\ &\quad \left. + (\vec{r}_1 + \vec{r}_4 - \vec{r}_2 - \vec{r}_3) \wedge (\vec{p}_1 + \vec{p}_4 - \vec{p}_2 - \vec{p}_3) \right\} \end{aligned} \quad (1)$$

where  $\vec{r}_i$  and  $\vec{p}_i$  stand for the coordinates (position and momentum) of the  $i$ -th quark. The first term on the right-hand side of Eq. (1) is the centre-of-mass angular momentum, while the three other terms are the relative angular momenta, commuting between each other. Their corresponding eigenfunctions have to be combined with the  $SU(6)_{FS}$  functions in order to satisfy a global symmetry conforming to the [3,1] Young tableau. Thus, composing with the color 3 wave function part, one will finally obtain a completely antisymmetric wave function for the  $4q$  state. For convenience, let us introduce the notation:

$$\vec{r}_{1i} = \frac{1}{2}(\vec{r}_1 + \vec{r}_i - \vec{r}_j - \vec{r}_k) \quad (2)$$

with  $i, j, k = 2, 3, 4$  all different. Considering as an example the exchange  $\vec{r}_1 \leftrightarrow \vec{r}_2$ , one notes that  $\vec{r}_{12}$  is unchanged, while  $\vec{r}_{13} \leftrightarrow -\vec{r}_{14}$ , implying analogous transformations on the spherical

harmonics:  $Y_{1m}(\vec{r}_{13}) \leftrightarrow -Y_{1m}(\vec{r}_{14})$ . Now, there are four different  $SU(6)$  irreducible representations which are present in the decomposition of the product  $6 \times 6 \times 6 \times 6$ , the representations 126, 210, 105 and 105' with corresponding Young tableaux [4], [31], [2<sup>2</sup>] and [21<sup>2</sup>] respectively, for which one can determine the totally antisymmetric wave function under  $SU(6)_{FS} \times O(3)_L \times SU(3)_C$ . The result is as follows:

$$Ant. \left\{ F_{126}(\vec{r}_i \cdot \vec{r}_j) \psi_{\{A,B,C,D\}}^{126} [Y_{1m}(\vec{r}_{12}) + Y_{1m}(\vec{r}_{13}) + Y_{1m}(\vec{r}_{14})] f_{[\beta,\gamma,\delta],\alpha} \right\} \quad (3)$$

$$\begin{aligned} Ant. \quad & \left\{ F^{210}(\vec{r}_i \cdot \vec{r}_j) \left[ (\psi_{\{A,C,D\},B}^{210} + \psi_{\{A,B,D\},C}^{210}) (Y_{1m}(\vec{r}_{12}) + Y_{1m}(\vec{r}_{13})) \right. \right. \\ & + \left. \left( \psi_{\{A,B,C\},D}^{210} + \psi_{\{A,C,D\},B}^{210} \right) (Y_{1m}(\vec{r}_{12}) + Y_{1m}(\vec{r}_{14})) \right. \\ & + \left. \left. \left( \psi_{\{A,B,C\},D}^{210} + \psi_{\{A,B,D\},C}^{210} \right) (Y_{1m}(\vec{r}_{13}) + Y_{1m}(\vec{r}_{14})) \right] f_{[\beta,\gamma,\delta],\alpha} \right\} \end{aligned} \quad (4)$$

$$\begin{aligned} Ant. \quad & \left\{ F^{105}(\vec{r}_i \cdot \vec{r}_j) \left[ \psi_{\{A,B\}\{C,D\}}^{105} Y_{1m}(\vec{r}_{12}) \right. \right. \\ & + \psi_{\{A,C\}\{B,D\}}^{105} Y_{1m}(\vec{r}_{13}) \\ & + \left. \left. \psi_{\{A,D\}\{B,C\}}^{105} Y_{1m}(\vec{r}_{14}) \right] f_{[\beta,\gamma,\delta],\alpha} \right\} \end{aligned} \quad (5)$$

$$\begin{aligned} Ant. \quad & \left\{ F^{105'}(\vec{r}_i \cdot \vec{r}_j) \left[ \psi_{\{B,C\},D,A}^{105'} Y_{1m}(\vec{r}_{14}) \right. \right. \\ & + \psi_{\{B,D\},C,A}^{105'} Y_{1m}(\vec{r}_{13}) \\ & + \left. \left. \psi_{\{C,D\},B,A}^{105'} Y_{1m}(\vec{r}_{12}) \right] f_{[\beta,\gamma,\delta],\alpha} \right\} \end{aligned} \quad (6)$$

where the  $SU(6)$  space and colour coordinates of the four quarks are denoted by  $(A, \vec{r}_1, \alpha; B, \vec{r}_2, \beta; C, \vec{r}_3, \gamma; D, \vec{r}_4, \delta)$ , the  $F$ 's are scalar functions in the variables  $\vec{r}_i \cdot \vec{r}_j$  including a renormalization factor, the  $Y_{1,m}$  are the spherical harmonics and the  $\psi$  are defined on  $SU(6)$  flavour-spin variables with the symmetry in the lower indices corresponding to the Young tableaux:

$$\{A, B, C, D\} \rightarrow \begin{array}{|c|c|c|c|} \hline A & B & C & D \\ \hline \end{array} \quad (7)$$

$$\{A, B, C\}, D \rightarrow \begin{array}{|c|c|c|} \hline A & B & C \\ \hline D & & \\ \hline \end{array} \quad (8)$$

$$\{A, B\}\{C, D\} \rightarrow \begin{array}{|c|c|} \hline A & B \\ \hline C & D \\ \hline \end{array} \quad (9)$$

$$\{A, B\}, C, D \rightarrow \begin{array}{|c|c|} \hline A & B \\ \hline C & \\ \hline D & \\ \hline \end{array} \quad (10)$$

respectively, where we first symmetrize on the rows and then antisymmetrize on the columns. The colour wave function  $f_{[\beta,\gamma,\delta],\alpha}$  corresponds to the Young tableau:

$$\begin{array}{|c|c|} \hline \beta & \alpha \\ \hline \gamma & \\ \hline \delta & \\ \hline \end{array} \quad (11)$$

where we first antisymmetrize on the column and afterwards symmetrize on the row. In such a way the expressions within the  $\{\}$  are antisymmetric with respect to the exchange of the three sets of variables  $(B, \vec{r}_2, \beta; C, \vec{r}_3, \gamma; D, \vec{r}_3, \delta)$ . Finally, the operator *Ant.* has the task of antisymmetrizing in the four variables. It acts in the following way, transforming the product

$$\begin{array}{|c|c|c|} \hline b & c & d \\ \hline a & & \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline \beta & \alpha \\ \hline \gamma & \\ \hline \delta & \\ \hline \end{array}$$

which is antisymmetric in the variables  $(b, \beta; c, \gamma; d, \delta)$  into the combination:

$$\begin{array}{c} \begin{array}{|c|c|c|} \hline b & c & d \\ \hline a & & \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline \beta & \alpha \\ \hline \gamma & \\ \hline \delta & \\ \hline \end{array} \\ + \quad \begin{array}{|c|c|c|} \hline a & c & d \\ \hline b & & \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline \alpha & \beta \\ \hline \delta & \\ \hline \gamma & \\ \hline \end{array} \\ + \quad \begin{array}{|c|c|c|} \hline a & b & d \\ \hline c & & \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline \alpha & \gamma \\ \hline \beta & \\ \hline \delta & \\ \hline \end{array} \\ + \quad \begin{array}{|c|c|c|} \hline a & b & c \\ \hline d & & \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline \alpha & \delta \\ \hline \gamma & \\ \hline \beta & \\ \hline \end{array} \end{array} \quad (12)$$

which is antisymmetric in the variables  $(a, \alpha; b, \beta; c, \gamma; d, \delta)$ . Note that in this last operation we have made the identification  $(a, b, c, d) \equiv (A, \vec{r}_1; B, \vec{r}_2; C, \vec{r}_3; D, \vec{r}_4)$ .

The decomposition of the different  $SU(6)_{FS}$  representations into  $SU(3)_F \times SU(2)_S$  for  $L = 1$   $4q$ -states is reported in Table 1, together with the resulting  $SU(3)_F \times SU(2)_J$  representations obtained by composing them with the  $(\bar{3}, \frac{1}{2})$  and the orbital momentum.

$4q \ SU(6)_{FS}$	$SU(3)_F \times SU(2)_S$	$4q\bar{q} \ L=1 \ SU(3)_F \times SU(2)_J$
$126=[4]$	$(15'=[4], 2)$ $(15=[3,1], 1)$ $(\bar{6}, 0)$	$(35 + 10, \frac{7}{2} + 2\frac{5}{2} + 2\frac{3}{2} + \frac{1}{2})$ $(27 + 10 + 8, \frac{5}{2} + 2\frac{3}{2} + 2\frac{1}{2})$ $(\bar{10} + 8, \frac{3}{2} + \frac{1}{2})$
$210 = [3,1]$	$(15', 1)$ $(15, 2 + 1 + 0)$ $(\bar{6}, 1)$ $(3, 1 + 0)$	$(35 + 10, \frac{5}{2} + 2\frac{3}{2} + 2\frac{1}{2})$ $(27 + 10 + 8, \frac{7}{2} + 3\frac{5}{2} + 5\frac{3}{2} + 4\frac{1}{2})$ $(\bar{10} + 8, \frac{5}{2} + 2\frac{3}{2} + 2\frac{1}{2})$ $(8 + 1, \frac{5}{2} + 3\frac{3}{2} + 3\frac{1}{2})$
$105=[2^2]$	$(15', 1)$ $(15, 0)$ $(\bar{6}, 2 + 0)$ $(3, 1)$	$(35 + 10, \frac{5}{2} + 2\frac{3}{2} + 2\frac{1}{2})$ $(27 + 10 + 8, \frac{3}{2} + \frac{1}{2})$ $(\bar{10} + 8, \frac{7}{2} + 2\frac{5}{2} + 3\frac{3}{2} + 2\frac{1}{2})$ $(8 + 1, \frac{5}{2} + 2\frac{3}{2} + 2\frac{1}{2})$
$105'=[2,1^2]$	$(15, 1 + 0)$ $(\bar{6}, 1)$ $(3, 2 + 1 + 0)$	$(27 + 10 + 8, \frac{5}{2} + 3\frac{3}{2} + 3\frac{1}{2})$ $(\bar{10} + 8, \frac{5}{2} + 2\frac{3}{2} + 2\frac{1}{2})$ $(8 + 1, \frac{7}{2} + 3\frac{5}{2} + 5\frac{3}{2} + 4\frac{1}{2})$

TABLE 1 : Representations of  $(4q)$  states and  $L = 1(4q + \bar{q})$  states. For convenience, the  $SU(2)$  representations are not denoted by their dimensions - which is the case for their  $SU(3)$  partners - but by their highest weight.

Let us add that a state constructed with four  $S$ -wave quarks and one antiquark with relative angular momentum  $L = 1$  is also in the 210 representation of  $SU(6)_{FS}$ . It therefore belongs to one of its corresponding  $SU(3)_F \times SU(2)_J$  multiplets expressed in Table 1, but, of course, does not share the same wave function as the one given by Eq.(4). All the four  $SU(6)_{FS}, L = 1$  representations considered in Table 1 contain a  $(\overline{10}, \frac{1}{2})$  multiplet of  $SU(3)_F \times SU(2)_S$ . However, the two last cases, i.e., the  $105, L = 1$ , which corresponds to the proper four-quark antisymmetrization of the two diquark model proposed in [6], and the  $105', L = 1$ , share a property [7] which strongly favours them for the classification of the experimentally found  $\Theta^+$  and  $\Xi^{--}$  states. Indeed, when the  $\bar{q}$  picks up one of the  $q$  to build a meson – in the case of  $\Theta^+$ , the  $\bar{s}$  together with a  $u$  or a  $d$  forms a  $K$  – two of the remaining three  $q$ 's remain in a  $SU(6)_{FS}$  antisymmetric state, so that this wave function is orthogonal to that of the totally symmetric 56  $SU(6)_{FS}$  representation: this implies a selection rule against the decay of the pentaquark into a meson plus a baryon of the  $1/2^+$  octet or of the  $3/2^+$  decuplet (let us recall the  $SU(3) \times SU(2)$  decompositions of the  $56 = (8, 1/2) + (10, 3/2)$ .) This property seems appealing to account for the narrowness of the discovered pentaquark states. Since the selection rule has been found in the limit of flavour symmetry, we expect it to be violated by  $SU(2)_I$ -breaking terms for  $\Theta^+ \rightarrow KN$  (as happens for  $\eta \rightarrow 3\pi$  decay) and by  $SU(3)_F$  breaking for  $\Xi^{--} \rightarrow \Xi^- \pi^-$ . There are many  $\bar{6}$   $SU(3)_F$  representations among the  $105' + 105, L = 1$  states which may build the  $\overline{10}$  together with an  $S$ -wave  $\bar{q}$ , where we wish to classify the  $\Theta^+$  and the  $\Xi^{I=3/2}$  states. To choose between them, one should get an even qualitative idea about the spectrum of these pentaquark states with the twofold motivation of finding a sufficiently low value for the mass of the  $\Theta^+$ , and further, with small components along the  $126 + 210, L = 1$ .

The mass differences within the  $S$  wave  $3q$  and  $q\bar{q}$  states are well reproduced by the QCD chromo-magnetic interaction [8]. The fact that the binding energies are given by combinations of the Casimir colour-spin  $SU(6)_{CS}$ ,  $SU(2)_S$  and  $SU(3)_C$  [9], allows us, as in the Arima and Iachello model for nuclear physics [10], to deduce the general properties of the pentaquark spectrum from group theoretical considerations. The two-quark interaction is attractive if their wave function is symmetric in colour and spin and repulsive in the other case. So the form of the Young tableaux for  $SU(6)_{CS}$  dictates the shape of the spectrum: for the states of the 56, the  $(10, 3/2^+)$ , which transforms as the totally antisymmetric 20 representation of  $SU(6)_{CS}$ , is heavier than the  $(8, 1/2^+)$ , which transforms as the 70 of  $SU(6)_{CS}$ , which has a mixed symmetry. For the negative parity states of the 70  $L = 1$ , one can explain the differences between the mass of the  $J^P = \frac{5}{2}^-$  resonances and the mean values of the masses of the  $J^P = \frac{3}{2}^-$  and  $\frac{1}{2}^-$  resonances with a  $LS$  coupling with a coefficient of  $40MeV$  and no contribution from the chromo-magnetic quark interaction ( we would rather expect a  $\frac{1}{3}$  screening factor from the orbital angular momentum rather than a complete one), which would follow from the fact that the states constructed with the  $(8, S = 3/2)$  transform as a 20 of  $SU(6)_{CS}$  and the states of the  $(10 + 8 + 1, S = 1/2)$  transform as a 70 under  $SU(6)_{CS}$ . Nevertheless, even in absence of an effect of the spin forces for the  $L = 1$  states of  $4q$  we are considering, it is useful to classify the  $SU(3)_F \times SU(2)_S$  multiplets listed in Table 1 according to their transformation properties with respect to  $SU(6)_{CS}$  in order to simplify the diagonalization of their interaction with the  $\bar{q}$ . As in the case of  $3q$  states, we find a spectrum, factorized in flavour and colour, as described in Table 2:

$SU(6)_{CS}$	$SU(3)_F \times SU(2)_S$
210	$(15 + \bar{6} + 3, S = 1 + 0)$
105	$(15 + 3, S = 1)$
105'	$(15' + 15 + \bar{6} + 3, S = 2 + 1 + 0)$
$\bar{15}$	$(15, S = 1)$

Table 2: Colour spin versus flavour and spin for  $L = 1$  colour triplet  $4q$  states.

For the interested reader, we summarize how we have constructed Table 2. We selected in Table 1 the  $SU(3)$  3 representations (in this case we think of colour, but group theory does not care about the interpretation) and we consider also the  $\bar{15}$  of  $SU(6)$ , which contains a  $(3, S = 1)$  multiplet. By considering the spin associated to the 3 of  $SU(3)$  of each  $SU(6)_{CS}$  representation, we find its spin content. To find the flavour content, we compose the Young tableau associated to each  $SU(6)_{CS}$  representation with the Young tableau  $[3,1]$  corresponding to the orbital momentum, and we consider the dual tableaux of the ones obtained (by dual of a Young tableau, we mean that obtained by composing with the diagram  $[1^4]$ ). To understand the form of the spectrum of the pentaquark states that can be built with the  $L = 1$   $4q$  and a  $\bar{q}$  in an  $S$ -wave with respect to them, it is useful to make an analogy with  $q\bar{q}$  states, showing the consequences of the QCD chromo-magnetic force for them. In fact, this force is strongly attractive [8] for  $SU(6)$  singlets, which is the case of the pseudoscalar mesons, and slightly repulsive for the colour singlet spin-1 states, i.e., the vector mesons, which are classified in a 35  $SU(6)_{CS}$  representation. For  $L = 1$   $q\bar{q}$  states, this force is screened by the orbital angular momentum, but in our case the  $\bar{q}$  is in a  $S$ -wave with respect to the  $4q$  states. The orbital momentum of these  $4q$  states is expected to screen the chromo-magnetic force by a factor which we guess to be  $1/2$ : it would correspond to the description of a  $\bar{q}$  clustered with one of a  $S$ -wave  $qq$  pair, as suggested in [9] many years ago and more recently considered in [11], but it is worth recalling that the approach used here is fully consistent with Fermi-Dirac quark statistics.

The lesson to be learned from the successful QCD description of the  $q\bar{q}$   $L = 0$  states is that the chromo-magnetic force is attractive for small  $SU(6)_{CS}$  representations, as dictated by the general formula, with a positive coefficient for the Casimir of the final  $SU(6)_{CS}$  representation. Therefore in the combination of the  $SU(6)_{CS}$  representations corresponding to the colour triplet  $4q$   $L = 1$  states described in Table 2, with the  $\bar{q}$ , we select the smallest possible  $SU(6)_{CS}$  representations, to get the lightest states of the spectrum. These representations are the 70 for 210 and 105, the 20 for 105' and  $\bar{15}$ ; in the case of the 105' we consider also the 70, for which the interaction is also attractive. As previously stated, the 20 and 70 of  $SU(6)_{CS}$  contain a colour singlet with spin  $3/2$  and  $1/2$ , respectively. So for the  $Y = 2$  pentaquarks with quark content  $uudd\bar{s}$ , we may write the phenomenological formula for the mass spectrum [9]:

$$m = m_0 + h \frac{3}{16} (m_{K^*} - m_K) \left[ C_6(p) - C_6(t) - \frac{1}{3} S_p(S_p + 1) + \frac{1}{3} S_t(S_t + 1) - \frac{4}{3} \right] + \tilde{h} \frac{1}{4} (m_N - m_\Delta) \left[ C_6(t) - \frac{1}{3} S_t(S_t + 1) - \frac{26}{3} \right] + a \vec{L} \cdot \vec{S} \quad (13)$$

where  $C_6(p)$  and  $C_6(t)$  are Casimir of  $SU(6)_{CS}$  representations, where the pentaquarks and the tetraquarks are classified, respectively, normalized as  $C_6(35) = 6$ . The  $K^{(*)}$  and the  $Y = 1$  baryons have been considered, since they involve the flavours relevant for  $Y = 2$  pentaquarks. According to our guess we take  $h = \frac{1}{2}$  and  $\tilde{h}$  and  $a$  from the spectrum of the 70  $L = 1$  with  $Y = 1$  to be 0 and  $40MeV$ , respectively. For the diagonalization of the mass spectrum given by eq.(13) it is useful to know the Clebsh-Gordon coefficients:

$$\begin{aligned}
& |20, (1, S = 3/2), S_z = 3/2 \rangle = \\
& \frac{1}{\sqrt{3}} \left\{ \frac{2}{\sqrt{7}} |105'; (3, S = 2), S_z = 2 \rangle_a \quad |\bar{6}; (\bar{3}, S = 1/2), S_z = -1/2 \rangle^a \right. \\
& \frac{-1}{\sqrt{7}} |105'; (3, S = 2), S_z = 1 \rangle_a \quad |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 \rangle^a \\
& \left. + \sqrt{\frac{2}{7}} |105'; (3, S = 1), S_z = 1 \rangle_a \quad |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 \rangle^a \right\} \quad (14)
\end{aligned}$$

$$\begin{aligned}
& |70, (1, S = 1/2), S_z = \frac{1}{2} \rangle = \\
& \frac{1}{\sqrt{3}} \left\{ \frac{1}{\sqrt{3}} |105'(3, S = 1)S_z = 1 \rangle_a \quad |\bar{6}; (\bar{3}, S = 1/2), S_z = -1/2 \rangle^a \right. \\
& \frac{-1}{\sqrt{6}} |105'(3, S = 1)S_z = 0 \rangle_a \quad |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 \rangle^a \\
& \left. + \frac{1}{\sqrt{2}} |105'(3, S = 0) \rangle_a \quad |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 \rangle^a \right\} \quad (15)
\end{aligned}$$

$$\begin{aligned}
& |70, (1, S = 1/2), S_z = \frac{1}{2} \rangle = \\
& \frac{1}{\sqrt{3}} \left\{ \frac{1}{\sqrt{2}} |210(3, S = 1)S_z = 1 \rangle_a \quad |\bar{6}; (\bar{3}, S = 1/2), S_z = -1/2 \rangle^a \right. \\
& -\frac{1}{2} |210(3, S = 1)S_z = 0 \rangle_a \quad |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 \rangle^a \\
& \left. + \frac{1}{2} |210(3, S = 0) \rangle_a \quad |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 \rangle^a \right\} \quad (16)
\end{aligned}$$

where  $a = 1, 2, 3$  is a colour index to be saturated to get a colour singlet.

In fact, by neglecting the dependance  $S_t$  in Eq.(13), the l.h.s.'s of eq.'s (14-16) would be eigenvectors of the mass. The exact consequences of Eq.(13) are reported in Table 3, where the mass of the  $Y = 2$  pentaquarks is given for each set of multiplets. The effects of  $SU(3)_F$  breaking will be dealt in a forthcoming paper [12].

$SU(6)_{CS} \times S$	$SU(3)_F \times SU(2)_J$	M (MeV)
$(20^*, \frac{3}{2})(105')$	$(35 + 10 + 27 + 10 + 8 + \overline{10} + 8 + 8 + 1, \frac{5}{2} + \frac{3}{2} + \frac{1}{2})$	$1640 + 40(\vec{L} \cdot \vec{S})$
$(70^*, \frac{1}{2})(210)$	$(27 + 10 + 8 + \overline{10} + 8 + 8 + 1, \frac{3}{2} + \frac{1}{2})$	$1600 + 40(\vec{L} \cdot \vec{S})$
$(70, \frac{1}{2})(105)$	$(27 + 10 + 8 + 8 + 1, \frac{3}{2} + \frac{1}{2})$	$1681 + 40(\vec{L} \cdot \vec{S})$
$(20, \frac{3}{2})(\overline{15})$	$(27 + 10 + 8, \frac{5}{2} + \frac{3}{2} + \frac{1}{2})$	$1755 + 40(\vec{L} \cdot \vec{S})$
$(70^*, \frac{1}{2})(105')$	$(35 + 10 + 27 + 10 + 8 + \overline{10} + 8 + 8 + 1, \frac{3}{2} + \frac{1}{2})$	$1742 + 40(\vec{L} \cdot \vec{S})$
$(540, \frac{5}{2})(105')$	$(35 + 10 + 27 + 10 + 8 + \overline{10} + 8 + 8 + 1, \frac{7}{2} + \frac{5}{2} + \frac{3}{2})$	$1854 + 40(\vec{L} \cdot \vec{S})$
$(1134, \frac{3}{2})(210)$	$(27 + 10 + 8 + \overline{10} + 8 + 8 + 1, \frac{5}{2} + \frac{3}{2} + \frac{1}{2})$	$1866 + 40(\vec{L} \cdot \vec{S})$
$(560, \frac{3}{2})(105)$	$(27 + 10 + 8 + 8 + 1, \frac{5}{2} + \frac{3}{2} + \frac{1}{2})$	$1866 + 40(\vec{L} \cdot \vec{S})$
$(540^*, \frac{3}{2})(105')$	$(35 + 10 + 27 + 10 + 8 + \overline{10} + 8 + 8 + 1, \frac{5}{2} + \frac{3}{2} + \frac{1}{2})$	$1882 + 40(\vec{L} \cdot \vec{S})$
$(1134^*, \frac{1}{2})(210)$	$(27 + 10 + 8 + \overline{10} + 8 + 8 + 1, \frac{3}{2} + \frac{1}{2})$	$1885 + 40(\vec{L} \cdot \vec{S})$
$(540^*, \frac{1}{2})(105')$	$(35 + 10 + 27 + 10 + 8 + \overline{10} + 8 + 8 + 1, \frac{3}{2} + \frac{1}{2})$	$1885 + 40(\vec{L} \cdot \vec{S})$
$(70, \frac{1}{2})(\overline{15})$	$(27 + 10 + 8, \frac{3}{2} + \frac{1}{2})$	$1903 + 40(\vec{L} \cdot \vec{S})$

Table 3: Mass spectrum of the positive parity pentaquarks built with  $4q$  with  $L = 1$  and a  $\bar{q}$  in S-wave. The \* is put to remind of a mixing between the  $SU(6)_{CS}$  representations and the transformation properties of the  $4q$  state have been written in brackets, since they are relevant for the  $SU(6)_{CS}$  selection rule, which is shown in the following lines.

We have a selection rule in  $SU(6)_{CS}$  analogous to the one found in the framework of  $SU(6)_{FS}$  [7]. This new selection rule comes from the fact that the  $(10, \frac{3}{2}^+)$  and the  $(8, \frac{1}{2}^+)$  transform as the 20 and 70  $SU(6)_{CS}$  representations, respectively. The states of the 210 and of the 105  $SU(6)_{CS}$  cannot decay into meson decuplet states, while the states of the  $\overline{15}$  cannot decay into meson octet states. Let us emphasize that in this case the selection rule is not affected by  $SU(3)_F$  and  $SU(2)_I$  breaking. In conclusion, only the states, with their  $4q$  transforming as the 105' of  $SU(6)_{CS}$  may be found by looking for decuplet-meson final states in octet-meson reactions.

The lightest multiplets are the  $J^P = \frac{1}{2}^+$  states built by combining the  $S = \frac{3}{2}$  state approximately given by Eq.(14) with the orbital momentum  $L = 1$ . In the  $\overline{10}$  we propose to classify the  $\Theta^+$  and the  $\Xi^{I=\frac{3}{2}}$  particles. For this reason we have fixed  $m_0$  in Table 3 to reproduce the mass of  $\Theta^+$ . In fact that  $\overline{10}$  has reduced couplings to MB final states as, which come by the product of three factors.

In fact the  $(\bar{6}, S = 2)$  multiplet is in the 105 of  $SU(6)_{FS}$  and obeys the selection rule preventing the decay into MB (i.e. meson-baryon) state. The  $(\bar{6}, S = 1)$  of the  $SU(6)_{CS}$  105', for which the exact computation upgrades the factor  $\sqrt{\frac{2}{7}}$  to  $\simeq \sqrt{\frac{3}{8}}$ , is an equal mixture of the  $SU(6)_{FS}$  210 and 105', which implies a reduction factor of  $\frac{1}{\sqrt{2}}$ . Another reduction factor  $\frac{1}{\sqrt{2}}$  comes by considering the SL tensor product  $\frac{3}{2} \times 1$ , which gives the total angular momentum  $\frac{1}{2}$  of the pentaquark. The  $S_z = \frac{3}{2}$  state appears with a factor  $\frac{1}{\sqrt{2}}$  in that pentaquark state and the component coming from the  $S=1$  state, which is the one, for which the decay into MB is allowed, has  $S_z^q = 1$  and  $S_z^{\bar{q}} = \frac{1}{2}$ . Therefore the  $\bar{q}$  to form a meson should take the quark with opposite  $S_z$ , leaving the



remaining quarks with  $S_z = \frac{3}{2}$  with the wave function orthogonal to the baryon octet ( the decay into the meson plus decuplet of a particle in a  $\bar{10}$  of  $SU(3)_F$  violates  $SU(3)_F$  symmetry). In conclusion we predict a global reduction factor of  $\frac{3}{32}$  for the width of the lightest decuplet state according to Eq.(13), which is welcome to explain the narrowness of  $\Theta^+$  and  $\Xi^{I=\frac{3}{2}}$  states. In the case of the  $\Xi^{--}$ , the narrow width of which is still more surprising due to the large phase space available for decay into  $\Xi^-\pi^-$ , there may be a destructive interference between the contribution of the 210  $SU(6)_{FS}$  state and the flavour-violating contribution, which one expects for the states of the  $105 + 105'$ .

The preliminary result, announced at the recent Jefferson Laboratory pentaquark conference, of a  $\Xi^*(1530)\pi$  resonance found by NA49 at a mass near to the  $\Xi^{--}$  (and  $\Xi^0$ ) found in  $\Xi^-\pi^-(\pi^+)$  final states with a comparable number of events above the background would be a striking confirmation of our proposal. Indeed, we expect amplitudes of the same order for the  $SU(3)$  allowed  $\Xi^{I=3/2}$  decays into  $\Xi\pi$  and for the  $SU(3)$  forbidden  $\Xi^{I=3/2} \rightarrow \Xi^*\pi$  decay (the product  $10 \times 8 = 35 + 27 + 10 + 8$  does not contain a  $\bar{10}$  representation)<sup>3</sup>.

We can develop for the 3's of  $SU(3)_F$ , transforming as the  $105'$  of  $SU(6)_{CS}$ , the same considerations as for the  $\bar{6}$ 's, leading to the conclusion of the existence of the multiplet of states ( $\bar{10} + 8 + 8 + 1, 5/2^+ + 3/2^+ + 1/2^+$ ) weakly coupled to the  $MB$  channels. The  $\bar{10} 1/2^+$  is the state, we have proposed for the classification of the  $\Theta^+$  and  $\Xi^{I=3/2}$  discovered pentaquarks. One might think that the three quarks left in a parity negative state by the S-wave emitted meson could produce a  $q\bar{q}$  pair to give a  $MB\pi$  final state. The  $\Theta^+$  state is below the threshold for producing a  $KN\pi$  state, but for  $\Xi^{I=3/2}$  it would be worth looking for  $\Xi\pi\pi$  final states.

The low couplings to MB final states of the  $(8 + 8 + 1, 1/2^+)$  multiplets, as well as those of the remaining states of the  $\bar{10}$ , which are classified in the same multiplet of  $\Theta^+$  and  $\Xi^{I=3/2}$ , explain why they succeeded up to now in escaping observation. They have the same  $I, Y$  quantum numbers of  $3q$  states strongly coupled to the meson-baryon channels, and the few events for which they are responsible are hidden by the overwhelming number of events due to  $3q$  resonant states. Only very accurate experiments, encouraged by the knowledge of their existence and made easier by some prejudice on the values of their masses, would reveal them. The other light multiplets built with the  $(15' + 15, S = 2)$  tetraquarks of the  $105'$  representation of  $SU(6)_{CS}$  have not reduced couplings to MB final states, since they transform as the 126 and 210 representations of  $SU(6)_{FS}$ , for which the flavour selection rule discovered in [7] does not apply. More in general all the states with large components along the 210 and 126  $L = 1$   $SU(6)_{FS}$  multiplets are allowed to decay into  $BM$  and not expected to be narrow. The Roper resonance may be classified as one of these states. In particular the 35 and 27 of  $SU(3)_F$  contain  $Y = 2, I = 2$  (to be looked for in the  $N^{*++}K^+$  final state) and  $I = 1$  states, respectively.

An inspection to the mass spectrum described in Table 3 shows that all the states lay below  $2 GeV$ , below the threshold to decay into  $K$  negative parity baryon states, to which the pentaquark particles here described are expected to have large couplings. However we expect some of their  $SU(3)_F$  partners to be above the threshold to decay into  $\pi$  negative parity baryon states.

We may apply Eq.(13) to the study of the spectrum of the negative parity states constructed with  $4q$  and a  $\bar{q}$  in a  $S$ -wave, we have mentioned at the beginning of this letter. We expect a

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<sup>3</sup>We thank M. Karliner for bringing this fact to our attention during his seminar at CERN.

spectrum with the  $3, \bar{6}, 15$  and  $15'$  representations with increasing mass, since they transform as the  $210, 105, 105'$  and  $\bar{15}$  representations of  $SU(6)_{CS}$ , respectively. From Eq.(13) with a different value of the central mass, we call for them  $\tilde{m}_0$ , and  $h = \tilde{h} = 1$ , since we do not expect the chromo-magnetic to be screened, we find the spectrum reported in Table 4.

$SU(6)_{CS}$	$SU(3)_F \times SU(2)_{J=S}$	M (MeV)
$70^*(210)$	$(8 + 1, \frac{1}{2})$	$\simeq \tilde{m}_0 - 669$
$70(105)$	$(\bar{10} + 8, \frac{1}{2})$	$\tilde{m}_0 - 345$
$20^*(105')$	$(27 + 10 + 8, \frac{3}{2})$	$\tilde{m}_0 - 222$
$1134(210)$	$(8 + 1, \frac{3}{2})$	$> \tilde{m}_0 - 123$
$70^*(105')$	$(27 + 10 + 8, \frac{1}{2})$	$\tilde{m}_0 - 107$
$560(105)$	$(\bar{10} + 8, \frac{3}{2})$	$\tilde{m}_0 + 25$
$540^*(105')$	$(27 + 10 + 8, \frac{1}{2})$	$\tilde{m}_0 + 205$
$540(105')$	$(27 + 10 + 8, \frac{3}{2})$	$\tilde{m}_0 + 247$
$540^*(105')$	$(27 + 10 + 8, \frac{3}{2})$	$\tilde{m}_0 + 247$
$20(\bar{15})$	$(35 + 10, \frac{3}{2})$	$\tilde{m}_0 + 247$
$1134^*(210)$	$(8 + 1, \frac{1}{2})$	$> \tilde{m}_0 - 322$
$70(\bar{15})$	$(35 + 10, \frac{1}{2})$	$\tilde{m}_0 + 543$

Table 4: Spectrum of negative parity states built with  $4q$  and a  $\bar{q}$  in S-wave. The notations are as in Table 3 and the mass of the  $Y = 2$  states are reported with the only exception of the  $8 + 1$  multiplets with  $4q$  transforming as the  $210$  of  $SU(6)_{CS}$ , for which the mass reported is the one of the  $Y = 0$   $I = 1$  state ( with quark content  $uuds\bar{d}$  ). The fact that there is a strange  $q$ , instead of a  $\bar{q}$  will make the numbers written in Table 4 for these states affected by  $SU(3)_F$  breaking, but the effect should be less relevant for the lightest state, for which the two relevant terms in Eq.(13) give comparable contributions, than for the other two multiplets: this motivates the different mathematical symbols present in Table 4.

The spectrum of the positive parity states built with  $4q$  in S-wave and a  $\bar{q}$  with  $L = 1$  with respect to them may be found by Eq.(13), by taking  $h = 0$ ,  $\tilde{h} = 1$  and  $a = 40MeV$ .

By applying Eq.(13) to  $qq\bar{q}\bar{q}$  meson states with  $h = \tilde{h} = 1$ , modified to keep into account that the chromo-magnetic interaction is slightly stronger for  $ud$  quarks than for  $s$  ( $m_\rho - m_\pi = \frac{4}{3}(m_{K^*} - m_K)$ ), one would obtain the intriguing result that the lightest state, with a contribution of the chromo-magnetic interaction  $\simeq -1GeV$ , is a  $I = 0$  state with quark content  $ud\bar{u}\bar{d}$ , which transforms as a singlet of  $SU(6)_{CS}$ , to be identified with the  $f^0(600) 0^+$  state. Several hundreds of MeV above that state one predicts a ( $I = 1 + 0, 0^+$ )  $qs\bar{q}\bar{s}$  multiplet to be identified with the  $f^0(980)$  and  $a_0(980) 0^+$  states, for which the  $qs\bar{q}\bar{s}$  content has been already proposed [13].

As suggested in [14], we should look for narrow  $\bar{D}N$  and  $BN$  resonances that we would obtain by substituting, in  $\Theta^+$ ,  $\bar{s}$  with  $\bar{c}$  or  $\bar{b}$ , respectively. The states with  $4q$ , one of which is heavy, forbidden by the  $SU(6)_{FS}$  selection rule to decay into a baryon heavy meson state (let us think of a  $N^{*P=-1}D$  bound state) are not expected to be narrow, because for them the channel meson heavy baryon is opened and the  $SU(6)_{FS}$  selection rule is not effective by the large  $SU(4)_F$  breaking.

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